Beyond the threshold: how electoral size-dependent uncertainty affects majority determination

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Abstract

The determination of a majority threshold in any voting system can be influenced by voters' attitudes towards uncertainty. Traditionally, a higher majority threshold is associated with a risk-averse attitude, serving as a means to protect against the tyranny of the majority. Moreover, the absence of ex-ante information regarding the likelihood of the voting outcome introduces a further layer of uncertainty, that of ambiguity, which motivates decision-makers to seek increased protection.

In this study, we first provide a thorough formalization of this theoretical prediction, relying on a second-order expected utility model with both risk and ambiguity aversion of the voter toward the voting lottery. Second, we experimentally test its predictions by integrating the majority threshold implication into traditional experiments for risk and ambiguity elicitation. Through a series of classroom experiments run on 2020-2023 (about 1,100 subjects in Italy & France), we analyze how individuals, placed under varying conditions of uncertainty, react to the determination of a barrier threshold. We find a strong correlation between the number of voters and the chosen quorum for a majority: as each subject is only aware of her own voting preference, expanding the electoral base results in a more ambiguous probability about the outcome. This favors more conservative behavior and results in an upward adjustment of the majority threshold.

Keywords: voting lottery, majority threshold, risk and ambiguity attitude, theory-driven experiment

JEL Codes: D72, D81, C91

1 Introduction

Voting is commonly recognized as a mechanism for individuals to express their preferences and contribute to collective decision-making. However, in certain situations, voters face the additional challenge of determining an appropriate threshold that ensures the representation of a decisive majority. The determination of such a majority threshold is influenced by various factors, including individual attitudes towards risk and ambiguity. Given the inherent uncertainty of electoral outcomes, voting can be likened to a lottery, where the probability of achieving victory is contingent upon the chosen threshold (Attanasi et al., 2017). The expected value of this "electoral lottery" is significantly influenced by the voting process rules. Our study specifically investigates a majority system as it is considered the most effective way to minimize the risk of tyranny (Buchanan & Tullock, 1965; Rea, 1969; Badger, 1972; Curtis, 1972).

Compared to a number of studies (Mayerson & Weber, 1993; Laslier, 2009) that traditionally assume voting rules as exogenously determined and beliefs about other voters' behavior are endogenous, we do the opposite by developing a model in which the majority rule is directly determined by the agents themselves and we investigate how this condition is influenced by voters' risk aversion and ambiguity. We find that as the risk and ambiguity attitudes of a subject increase, they tend to choose a higher majority threshold, as a means to reduce the potential for dissatisfaction or contested outcomes.

However, a voter's decision-making process cannot overlook the impact of the electorate's size. In our study, we carefully examined subjects' behavior across 25 sessions, each one characterized by a distinct electoral base (min of 6 to a max of 280). Through a detailed analysis of individual responses and subsequent aggregation, we observed a consistent pattern: subjects exhibited a positive response as the size of the electorate increased. The size-dependent uncertainty arises due to statistical variations and inherent unpredictability in voter behavior. Hence, when the electoral size is small, the impact of uncertainty is more pronounced, and determining a clear majority can be challenging. In larger electorates, such as those in national or statewide elections, the law of large numbers comes into play, which decreases error provision as the number of voters grows. We show that as the electoral base becomes greater voters perceived a higher degree of uncertainty due to unpredictable actions about others. Consequently, they prefer to protect themselves against an unrepresentative political power and search for a higher quorum. We expect, and this is the new of our model, that the trend towards higher quorums in broader electoral contexts will be driven by ambiguity aversion, and not necessarily risk aversion.

The work is structured as follows. In Section 2 we set up our theoretical model and predictions. In Section 3 we describe our experimental design and in Section 4 we present the general results of our analysis.

2 Theoretical Framework

Political choices inherently involve decision-making under uncertainty. In this section, we introduce a straightforward theoretical model in which agents' preference for the majority threshold depends on their aversion to uncertainty.

2.1 The model

Consider a political system with N agents, where N is odd, and two alternative policy proposals, X and Y, for which they have to cast a ballot. Let q be an integer number denoting the majority rule, which lies between the simple majority and the unanimity. The choice of the majority rule is a choice under uncertainty because the voting outcome (winning, losing, status quo) is uncertain. The voting outcome is uncertain because voter i does not know how others will vote. Let n be the number of individuals will vote like i: n is unknown. The objective probability of n is unknown. Since the realisation of the voting outcome depends on the realisation of n, the objective probability of voting outcome is unknown. We refer to the latter situation as ambiguity. The probability of n is subjective. Suppose voter i thinks (subjectively) the probability another voter will vote for her own policy is p, with p ∈ (0,1), and the probability another voter will vote for the alternative policy is (1 – p). Since the other voters are in total N – 1, any other voter behaves as a Bernoulli random variable, say j, which takes 1 with probability p if she will vote like voter i; j takes 0 with probability (1 – p) if she will vote for the alternative policy. Therefore, n behaves as N – 1 independent Bernoulli random variables. That is, n is a discrete random variable that follow a binomial distribution, which takes N possible values, going from 0 to N – 1, with the following probability:

$$\mathrm{P(n)} = \begin{cases} \binom{N-1}{n} p^n (1-p)^{N-1} & \text{for n = 0, 1, ... , N-1} \\ 0 & \text{otherwise} \end{cases}$$

Since the probability of n is subjective, the probability of the voting outcome is subjective. In effect, Attanasi et al. (2014a) develop a behavioural model of majority threshold determination relying on a standard subjective expected utility model (Savage 1954). Therefore, the unknowledge of the objective probabilities of voting outcomes is assumed to be irrelevant to the voter i's choice of q. In other words, the choice of q under ambiguity is the same as the one under known objective probabilities, i.e. under risk. That is, by assumption, voter i is ambiguity neutral. As a result, Attanasi et al. (2014a)'s model accounts for risk aversion only. Our model extends their model by relaxing the assumption of ambiguity neutrality. The aim is to account also for ambiguity aversion. Our approach to uncertainty is to model ambiguity as multiple objective probabilities of voting outcomes. Precisely, for each voting outcome, there is an objective probability for each value of n. Our idea is to assume that, for each voting outcome (winning, losing, status quo), voter i has one belief for each possible objective probability. Note that, for each value of n, there is an objective probability distribution over voting outcomes. Therefore, the idea is to assume that voter i has one belief for each possible objective probability distribution. Such beliefs are called second-order probabilities (Neilson 1993). At the end of the day, we model ambiguity as set of second-order probabilities, whose cardinality, in this specific problem, is equal to the total number of voters, N. Then, we define ambiguity aversion as aversion to second-order probabilities. Consistently with standard risk theory, our model separates ambiguity aversion from ambiguity. Ambiguity aversion is captured by the concavity of a certain function ϕ. In particular, for any given q, voting is modelled as a compound lottery in which there are N possible states of the world, represented by the possible values of n. The model is an "expected utility over expected utilities", i.e. second-order expected utility model. The idea is that, in the evaluation of q, voter performs two steps: first, she computes the expected utility over voting outcomes for each possible state n; next, she computes the expectation (with respect to second-order probability) of the expected utilities obtained in the first step, each expected utility transformed by another function, ϕ. Note that, for each possible state n, it is possible to say a priori which voting outcome occurs. This implies that, for each n, the objective probability distribution over voting outcomes is degenerate. Voter i's second-order expected utility from the compound voting lottery is described as follows. Let EU_n be the expected utility function over the objective probability distribution over voting outcomes of state n. Let C denote the set of voting outcomes, with typical element x $\in$ C. We have C=$\{$L, S, W$\}$, with L representing "losing", S denoting "status quo", and W representing "winning". Let L, S, W $\in \Re$, with W $>$ S $\geq 0 >$ L and S $-$ L $=$ W $-$ S. Let u be a von Neumann-Morgenstern (1953) utility function over the set of (voting) outcomes, i.e. u:C$\rightarrow\Re$. Let U denote the range u(x):x$\in$C of u. Let ϕ be a continuous and increasing function ϕ:U$\rightarrow\Re$ such that EU_q=$\sum_n^{N-1} P(n)\phi[EU_n]$=$\sum_n^{N-1} P(n)\phi$[u(x)], where n = 0, . . . , N-1, second-order probabilities are given by the probability distribution of n, P(n), and for each n, EU_n = u(x). In this respect, voter i knows that:

u(x) = u(W) if n $\geq$ q $-$ 1;

u(x) = u(L) if n $<$ q $-$ 1 and N $-$(n +1) $\geq$ q;

u(x) = u(S) if n $<$ q $-$ 1 and N $-$(n +1) $<$ q.

Hence, we can rewrite the model as

$EU_q = \sum_{n=0}^{(N-1)-q} \binom{N-1}{n} p^n$(1-p)$^{(N-1)-n}\phi$[u(L)]+$\sum_{n=N-q}^{q-2} \binom{N-1}{n} p^n$(1-p)$^{(N-1)-n}\phi$[u(S)]+$\sum_{n=q-1}^{N-1} p^n$(1-

p)$^{(N-1)-n}\phi$[u(W)]

Note that both u and ϕ (risk and ambiguity attitudes) are state independent. Therefore, normalizing the status quo utility to zero (u(S) $\equiv$ 0), we can rewrite:

$$EU_q = \phi[u(L)] \sum_{n=0}^{(N-1)-q} \binom{N-1}{n} p^n (1-p)^{(N-1)-n} + \phi[u(W)] \sum_{n=q-1}^{N-1} p^(1-p)(N-1)-n. \quad (1)$$

Observe that all probabilities in EU_q depend on the voter i's quorum q. As a consequence, the expected value of compound voting lottery depends on q. Furthermore, note that as function ϕ is linear, the model boils down to the one of Attanasi et al.(2014a)[1]. Hence, consistently with standard risk theory, the shape of ϕ captures ambiguity attitudes in the following way: concavity of ϕ is equivalent to ambiguity aversion; convexity of ϕ is equivalent to ambiguity loving; so, linearity of ϕ is equivalent to ambiguity neutrality.

Given the expected utility model in (1), we study voter i's preference on majority thresholds as a function of her ambiguity aversion. In fact, our model studies voter i's preference on majority thresholds as a function of both ambiguity and risk aversion. However, risk aversion becomes relevant only if ϕ is linear, i.e. voter i is ambiguity neutral. Therefore, we study majority threshold determination as a function of uncertainty aversion by imposing a hierarchical order between ambiguity aversion and risk aversion. This implies that, as voter react to ambiguity, all of uncertainty is ambiguity and her attitudes towards risk become irrelevant. Now, the intuition underlying the relation between the preferred majority threshold and uncertainty aversion is that uncertainty aversion determines a higher (than simple majority) threshold. Indeed, since the expected value of the voting lottery does depend on the majority threshold, voter prefers the threshold which reduces voting outcome uncertainty. As Attanasi et al. (2014a), our model is behavioural because uncertainty aversion is determined by the fear of losing and being subject to a majority tyranny. As a result,

[1]That is, to "agent j's expected utility from the voting lottery: EU_j (Λ) = Pr{W}u_j(W) + Pr{L}u_j(L) + Pr{S}u_j(S)", where Λ = (W, Pr{W}; L, Pr{L}; S, Pr{S}) is the simple voting lottery (Attanasi et al., 2014a, equation 4, page 362).

individual prefers the rule which reduces the (un)certainty of losing. In fact, we introduce another psychological distortion on subjective probability p. Indeed, p can be interpreted as capturing voter i's degree of confidence regarding how the other N – 1 voters will vote. Hence, we define confidence attitudes as follows; p = 0.5 if voter i is unbiased; p < 0.5 if voter i is pessimist; p > 0.5 if voter i is optimist. At this point, we recall equation in (1) and we use the following trick: divide both sides of this equation by ϕ[u(W)]. Then, recalling u(S) ≡ 0, we have:

$$\frac{(EU_q)}{\phi[u(W)]} = \sum_{n=q-1}^{N-1} \binom{N-1}{n} p^n (1-p)^{(N-1)-n} - R_i \sum_{n=0}^{(N-1)-q} \binom{N-1}{n} p^n (1-p)^{(N-1)-n} \qquad (2)$$

where $R_i \equiv \frac{\phi[u(S)]-\phi[u(L)]}{\phi[u(W)]-\phi[u(S)]} > 0$.

We call R_i the ratio between the advantage of not losing (i.e. keeping the status quo) with respect to losing, ϕ[u(S)] - ϕ[u(L)], and the utility gain associated with winning with respect to keeping the status quo, ϕ[u(W)] - ϕ[u(S)]. This ratio measures the relative advantage of the status quo (Attanasi et al., 2014a).

Remark. By definition, if voter i is ambiguity averse, then the expected advantage of not losing is greater than the expected benefit of winning, i.e. ϕ[u(S)]-ϕ[u(L)] > ϕ[u(W)]-ϕ[u(S)]. For this reason, ratio $R_i > 1$ if voter is ambiguity averse. It is plausible to think that the more voter is scared about losing, the more she is uncertainty averse, the more the advantage of not losing is greater than the expected benefit of winning. As a result, R_i is positively related to voter i's degree of uncertainty aversion. We interpret R_i as an index of the degree of uncertainty aversion.

It only remains to study voter i's preferred majority threshold q as function of her uncertainty aversion. To the purpose, we imagine that voter i prefers the value of q which maximises her expected utility EU_q. Recall N is odd, so the majority threshold, which lies between the simple majority and the unanimity, is given by q ∈ [$\frac{N+1}{2}$, N]. Precisely, we imagine that voter i compares EU_q and EU_{q+1} to choose the preferred majority threshold.

Intuitively, she prefers q over q + 1 if and only if $EU_{q+1} \leq EU_q$, and vice-versa. In particular, voter i repeats the comparison between EU_q and EU_{q+1} for each value of q, from (q,q+1) = ($\frac{N+1}{2}$, $\frac{N+1}{2}$+1) to (N-1,N). If $EU_{q+1} \leq EU_q$ for all (q,q+1) then, by transitivity, the simple majority $\frac{N+1}{2}$ is preferred to all higher values of q. Conversely, for $EU_{q+1} \geq EU_q$ for all (q,q+1) then, by transitivity, the unanimity N is preferred to all lower values of q. To determine which of these two statements is true an ancillary result is used, which is based on Δ(q), the ratio between winning and losing probabilities (Attanasi et al., 2014a). In particular, Δ(q) is the ratio between the belief of the pivotal winning scenario n = q-1, say P_W (q-1), and the belief of the pivotal losing scenario n = (N –1)-q, say P_L (q). This ratio measures the relative belief of winning in the pivotal scenarios. That is[2]:

$$\Delta(\mathrm{q}) = \frac{P_W(q-1)}{P_L(q)} = \frac{q}{N-q}\left(\frac{p}{1-p}\right)^{2q-N}$$

Given Δ(q), we present the following twofold result[3]:

(a) EU is decreasing in q, i.e $\forall$ q $EU_{q+1} \leq EU_q \iff \forall$ q $\Delta(q) \geq R_i$;

(b) EU is decreasing in q, i.e $\forall$ q $EU_{q+1} \leq EU_q \iff \forall$ q $\Delta(q) \geq R_i$.

In fact, this result shows the main intuition underlying the majority threshold determination: the solution depends on voter's confidence attitudes, embedded in the relative probability of winning Δ(q), and voter's uncertainty attitudes, captured by R_i.

We study voter i' s preferred majority threshold q as function of her uncertainty aversion ceteris paribus. In order to keep confidence attitudes fixed, suppose voter i is unbiased, so p = 0.5. This implies that $\Delta(q) = \frac{q}{N-q}$. Note that, in this case, $\Delta(q) > 1$, for q $\in [\frac{N+1}{2}$, N]. Then, suppose voter i is ambiguity averse, so $R_i > 1$. Thus, for a sufficiently large R_i, voter i may have $\forall$ q $\Delta(q) \leq R_i$, from which the preferred majority threshold is the unanimity. This result does match with our initial intuition. Indeed, the idea is that an ambiguity-averse voter uses the majority threshold in order to reduce the uncertainty of losing. If unanimity applies, the uncertainty of losing is reduced to impossibility. Note again that, regardless

[2]This result can be proved easily. The "proof" is available on request from the authors to any interested reader.

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of confidence attitudes (p=0.5), the preferred majority threshold increases as the size of R_i increases.

2.2 Predictions

Since the sense of R_i is a proxy of the degree of ambiguity aversion, we derive the following result, ready to be tested:

Hypothesis 1. *Regardless of confidence attitudes, the higher the degree of ambiguity aversion the higher the preferred majority threshold.*

Two additional comments are worthy of mention. Firstly, symmetrically to ambiguity aversion, if voter i is ambiguity loving, the expected advantage of not losing is lower than the expected benefit of winning, i.e. ϕ[u(S)] - ϕ[u(L)] < ϕ[u(W)] - ϕ[u(S)]. In this case, one may assume ambiguity loving is determined by the hope of winning. This implies $R_i < 1$. Then, the preferred majority threshold of an unbiased ambiguity-loving voter is simple. If, given p = 0.5, Δ(q) $\geq R_i$, from which the optimal majority threshold is the simple majority. Secondly, if voter i is ambiguity neutral, then $R_i = \frac{u(S)-u(L)}{u(W)-u(S)}$. Since risk attitudes are emotion-dependent in exactly the same way as ambiguity attitudes, we have $R_i = \frac{u(S)-u(L)}{u(W)-u(S)} > 1$ if voter i is ambiguity neutral and risk averse; $R_i < 1$ if voter i is ambiguity neutral and risk loving; so,$R_i = 1$ if and only if voter i is ambiguity neutral and risk neutral. It can be easily checked that if an unbiased voter is ambiguity neutral, results from ambiguity aversion/loving exactly extend to risk aversion/loving. That is, regardless of confidence attitudes (p=0.5), the preferred majority threshold of an ambiguity-neutral and risk-averse voter, for $R_i = \frac{u(S)-u(L)}{u(W)-u(S)}$ sufficiently greater than 1, is the unanimity; the preferred majority threshold of an ambiguity neutral and risk loving voter is the simple majority. Thus, regardless of confidence attitudes, there should be a positive correlation between the degree of risk aversion and the preferred majority threshold only if voters are ambiguity neutral. Finally, we furthered the analysis by adding a relation between the total number of voters and the preferred majority threshold. The number of voting participants is used as an index of ambiguity perception. In order to embed such a relation into the model, we perform the following steps. First, we imagine N to be like an item, namely a bad, for an ambiguity-averse voter. Second, we make a concave transformation of function ϕ so as to make ϕ negatively dependent on N. We do so by using a specific transformation[4]:

$$\hat{\phi} = \begin{cases} \phi(\mathrm{u(x)})^{\frac{1}{N}} \text{if } \phi(\mathrm{u(x)}) > 0 \\ \phi(\mathrm{u(x)})^{N} \text{if } \phi(\mathrm{u(x)}) < 0 \end{cases}$$

Recalling u(S) $\equiv$ 0, we can rewrite model (2) as dependent on size N. That is: $\frac{EU_{q,N}}{\hat{\phi}[u(W)]} = \sum_{n=q-1}^{N-1} \binom{N-1}{n} p^n (1-p)^{(N-1)-n} - R_{N,i} \sum_{n=0}^{(N-1)-q} \binom{N-1}{n} p^n (1-p)^{(N-1)-n}, where \mathrm{R}_{N,i} \equiv \frac{-\hat{\phi}[u(L)]}{\hat{\phi}[u(W)]}$ $= \frac{-\hat{\phi}[u(L)]^N}{\hat{\phi}[u(W)]^{\frac{1}{N}}} > 0.$

It can be checked that $R_{N,i} > 1$ and it depends positively on N[5]. Since, for any N, $R_{N,i}$ is a proxy of the degree of ambiguity aversion, we derive the following result, ready to be tested:

Hypothesis 2. *Regardless of confidence attitudes, the higher the number of voters N the higher the degree of ambiguity aversion.*

Now, since the degree of ambiguity aversion is positively related to the preferred majority threshold, by transitivity, the relation between the electorate size N and the degree of ambiguity aversion closes the circle.

Hypothesis 3. *Regardless of confidence attitudes, the higher the number of voters N, the higher the preferred majority threshold.*

Remark. If voter is ambiguity neutral then index $R_i = \frac{u(S)-u(L)}{u(W)-u(S)}$, a proxy of the degree of risk aversion, is independent of the electorate size N.

[4]In doing so, assume that ϕ(u(L)) < -1, ϕ(u(W)) > 1. Moreover, assume ϕ(u) < 0 if and only u < 0. Then, assume ϕ(u) > 0 if and only u > 0.

[5]The "proof" is available on request from the authors to any interested reader.

3 Experimental design and procedures

To test our theoretical predictions we run modified versions of some traditional experiments for risk and ambiguity elicitation. The innovation is in the introduction of the elements for the majority threshold derived from the theoretical model presented in Section 3.

Experiments were conducted in 26 classes of graduate and undergraduate students between Italy (15 classes) and French (11 classes) from March 2020 to May 2023. The sample is composed of 1091 students (716 undergraduates and 375 graduates). Each one has participated in all three classroom experiments, which were conducted using Google Forms.

3.1 Classroom experiment 1

The first classroom experiment is a simplified replication of Kahneman and Tversky's (1979) lottery game for risk elicitation. Participants were presented with 12 problems, each representing a lottery with two possible outcomes: one outcome was certain or highly probable, while the other was uncertain, possible but not probable [6]. For each problem, they have to make a choice between the two alternatives. We will use the subject's choice in each problem as a proxy of his degree of risk aversion. To determine the final earnings, a computerized random draw was conducted, selecting one of the twelve problems. Each participant received the exact outcome they had chosen in that particular lottery, according to the associated probability.

3.2 Classroom experiment 2

The second classroom experiment aimed to elicit participants' risk aversion using different methods than the one mentioned earlier and investigate its relationship with voting choices[7]. The experiment was divided into three distinct phases.

[6]For detailed information see Kahneman and Tversky (1979).

[7]We assume as a reference to conduct our experiment what is done in Attanasi et al. (2014a)

In the first phase, participants engaged in a variant of the risk aversion elicitation mechanism developed by Holt and Laury (2002) - henceforth HL. Participants were presented with a battery of 19 pairs of lotteries, labeled from line L1 to line L19, with an additional empty line, L20. Each pair described two lotteries: one favorable and one no, with associated probabilities and potential constant monetary outcomes. Participants were tasked with choosing the line from which they preferred lottery B to lottery A. By analyzing participants' switching choices in this lottery game, their degree of risk aversion could be isolated. Specifically, a higher switching line indicated a higher degree of risk aversion. The final earnings in this phase were determined through a computerized random draw of one of the 19 pairs of lotteries. Based on the participant's choice and the pair selected by the computer, each participant took part in the preferred lottery. Subsequently, a second randomly drawn between 1 and 20 determines the outcome of the preferred lottery and the corresponding payoff for each participant.

The second phase of this classroom experiment is a variant of the original lottery-panel task of Sabater-Grande & Georgantizis (2002) - henceforth SG-G. Participants were presented with four decision problems, each involving a panel of ten two-outcome lotteries arranged in three rows. The first row presented the probabilities associated with a positive outcome, denoted as X, which were displayed in the second row. The third row consisted of ten spaces, allowing participants to select their preferred lottery from the available options in each of the four panels. For each panel of lotteries, each subject was asked to choose one of the ten lotteries (X, p) that implied a probability p of earning X, or else nothing. This means selecting, for each of the four panels, the preferred lottery among the ten available. In each panel, the selected lottery helps us to interpret the risk attitude of the subject. The final earnings in this phase were determined through two random draws. Firstly, one of the four panels was randomly selected, and subsequently, another draw isolated one of the ten lotteries within that panel. Participants had the opportunity to win the outcome associated with the drawn lottery if it aligned with their chosen preference. Otherwise, they would receive no earnings.

The third phase of the experiment introduced an innovative approach compared to the traditional methods used in the earlier phases. Each participant was randomly assigned an elector type, either X or Y, with an equal probability. Based on their assigned type, participants' votes were automatically allocated to the corresponding alternative: x-type voters supported alternative X, while y-type voters alternative Y. Following this voting process, participants were then asked to choose a majority threshold for a voting procedure between the two alternative types, ranged from a simple majority to unanimity. Additionally, they had to make an estimation regarding the distribution of x and y-type voters within the session. The final payoff is determined by comparing the subjects' chosen majority threshold with the actual distribution of x and y-type voters in the session. Specifically, if a participant's stated majority threshold was equal to or smaller than the number of participants of their own type, they would earn 22.00 euros. On the other hand, if their stated majority threshold was equal to or smaller than the number of participants of the other type, they would earn nothing. In cases where neither the number of participants of their own type nor that of the other type exceeded their stated majority threshold, participants would earn 11.00 euros. Additionally, if a participant correctly guessed the distribution of elector types, they would receive an additional 3.00 euros added to their earnings. Finally, the last part of this phase consists of a subject' self-assessment of his general attitude towards risk (Doheman et al., 2011)[8].

3.3 Classroom experiment 3

The third and last classroom experiment aims to elicit individuals' ambiguity aversion and pessimism attitudes in order to understand the impact on the choosing majority threshold. The experiment consists of four decision stages, and before it starts each participant has to

[8]The question was presented using the wording from Bernasconi et al. (2014): "In a scale from 1 to 10, how would you rate your attitude towards risk: are you a person always avoiding risk or do you love risk-taking behavior?", where 1 was associated with the statement "I always choose the safest option and try to avoid any possible risk" and 10 referred to "I love risk and I always choose the more risky alternative".

choose a letter between X or O. In Decision Stage 1 there are four lotteries (A, B, C. D) with known probabilities and outcomes and everyone has to choose one of them. If this stage is randomly drawn at the end of the experiment the randomly drawn letter gains the higher outcome associated with the chosen lottery while the other the smaller one. In Decision Stage 2 there is a unique lottery with certain outcomes and probability. Participants were asked to set a selling price for the lottery, which is between the minimum winning outcome and the maximum one (from 4 to 34). The mechanism implemented is the second-price auction, which means that, given a random draw from PC to determine the bid, if it is lower than the lowest computer bid, you win the auction, sell the lottery and receive as a price the lowest computer bid; if it is lower than the lowest computer bid, you lose the auction and play the lottery. Decision Stage 3 is exactly equal to Decision Stage 2 as well as Decision Stage 4 is equal to Decision Stage 1 but with the difference of unknown probabilities. Finally, for the stages with obscure probabilities, there is the elicitation of belief about the number of tickets having the same letter as the one chosen at the beginning of the experiment. This helps us to investigate the degree of pessimism/optimism of our subjects and study how this impact in their choice for a majority threshold.

Specifically, subjects' ambiguity attitude is elicited using Attanasi et al. (2014b) coherent-ambiguity attitude definition, which is considered as the combination of choice-ambiguity and value-ambiguity attitude.

Definition 1. (Value-ambiguity Attitude) Given CE(L_t) the subject's selling price assigned in stage t $\in$ {2,3}, a subject is value-ambiguity averse if CE(L_3) $\leq$ CE(L_2), value-ambiguity neutral if CE(L_3) = CE(L_2) and value-ambiguity loving if CE(L_3) $\geq$ CE(L_2).

Definition 2. (Choice-ambiguity Attitude) Given $L_t \in$ { A, B, C, D } the index of chosen lottery in stage t $\in$ {1,4}, a subject is choice-ambiguity averse if $L_4 \leq L_1$, choice-ambiguity neutral if $L_4 = L_1$ and choice-ambiguity loving if $L_4 \geq L_1$.

Definition 3. (Coherent-ambiguity Attitude) A subject is coherent-ambiguity averse if CE(L_3) $\leq$ CE(L_2) and $L_4 \leq L_1$, with at least one of the two relations holding strictly. A subject is coherent-ambiguity neutral if CE(L_3) = CE(L_2) and $L_4 = L_1$, while he is coherent-ambiguity loving if CE(L_3) $\geq$ CE(L_2) and $L_4 \geq L_1$, where one of the two conditions holding strictly.

4 Experimental results

In this section, we present our experimental results. Subsections 4.1 and 4.2 demonstrate how risk and ambiguity aversion contribute to an increase in the desired majority threshold. However, it is in subsection 4.3 that we delve into the most crucial analysis. We compare the effects of ambiguity and risk on voting behavior, specifically focusing on the growth of the electorate. Our findings reveal that as the electorate expands, the impact of ambiguity becomes more pronounced in driving the increase in the sought-after majority threshold.

4.1 Risk attitude and Majority threshold

Propensity and aversion risk traditional assumptions are confirmed. Classroom experiment 1 results confirm Kahneman & Tversky's prospect theory. Specifically, individuals exhibit risk aversion when it comes to potential gains, meaning they are more likely to choose a certain outcome over a risky lottery with a higher expected value [9]. Conversely, individuals may exhibit risk-seeking behavior when facing potential losses, being more inclined to take a risky option in the hope of avoiding losses [10]. Moreover, from a between-subjects analysis of experiment 2, 78% of subjects in HL[11] and 82.49% in SG-G (on average over the four panels)[12] disclose risk aversion. In a within-subjects, the number of individuals who choose to switch at L11 and then choose p = 40 % in Panel 1 and p = 20 % from Panel 2 - Panel 4 are 43.63 %. Specifically, the 40.5% of these subjects always choose the safest option in the first experiment. Additionally, in the third experiment 82.31% in Decision Stage 1 and

[9] In fact, respectively 50.87% and 70.85% in problems 1 and 3 choose the safest option.
[10] Not by chance 78.09% in problem 9 and 61.23% in problem 10 of subjects do this choice.
[11] The median choice is L16, revealing a higher degree of risk aversion.
[12] The frequencies are: 67.37% in Panel 1, 92.67% in Panel 2, 93.58% in Panel 3 and 76.35% in Panel 4.

56% in Decision Stage 2 are risk-averse subjects, and 48.58% show a high degree of risk aversion in both cases. By aggregating the results of all three experiments, it is evident that 24.47% of the subjects maintain a core of risk aversion in all phases. Furthermore, we find a significant and positive correlation between the HL index for risk aversion and the average one of SG-G (ρ = 0.157, p-value = 0.000) as well as between HL risk-averse index/SG-G average risk-aversion index and the self-assessment of risk (p-values = 0.000).

Given the degree of risk aversion, we find its impact on the choices for a majority threshold and confidence about other voters' behavior. First, we observe that in the guess about how other votes: 71.22% are overconfident (of these 7% are unbiased) and 28.78% unconfident. According to a Spearman correlation test, we find that there is no significant correlation between the degree of risk aversion (isolated in the variate HL lottery) and the confidence about how others will vote ($\rho = 0.022$, p-value = 0.465). In fact, the majority of subjects that exhibit risk aversion (about 70%) report overconfidence guesses. Thus, it can be observed that an overconfident subject, despite having a certain level of risk aversion, will never opt for a higher threshold compared to a less confident subject with the same level of risk aversion.

Table 1 reports the distribution of subject choices regarding thresholds in the third phase of the second experiment. In general, the median preferred threshold is around 68% and the modal threshold is unanimity.

Table 1. Distribution of majority thresholds

Majority threshold	n<30	30≤n<60	60≤n<100	n≥100	Average choice
50%	0.00	0.01	0.12	0.14	0.07
50% - 60 %	0.25	0.26	0.21	0.13	0.21
60% - 70 %	0.28	0.31	0.23	0.22	0.26
70% - 80 %	0.16	0.16	0.20	0.20	0.18
80% - 90 %	0.12	0.12	0.08	0.09	0.10
90% - 100%	0.06	0.03	0.06	0.07	0.06
100%	0.13	0.11	0.10	0.15	0.12

Comparing the degree of risk aversion previously elicited with the choice of the majority quorum, it becomes clear that, regardless of the size of the electoral base, a higher degree of risk aversion is associated with a higher majority threshold. This holds both for HL degree of risk aversion ($\rho = 0.100$, p-value = 0.00094) and average SG-G one ($\rho = 0.169$, p-value = 0.000). A positive and higher correlation is confirmed by Tobit regression, with a significance at 0.01%. Moreover, the Wilcoxon-Mann-Withney test suggests that the majority threshold chosen by risk-averse subjects is higher than those chosen by risk-loving subjects (p-value = 0.028). Therefore, those who are more risk-averse tend to exhibit a cautious approach and may prefer conservative voting strategies.

4.2 Ambiguity attitude and Majority threshold

Given the four decision tasks of experiment 3, subjects can be classified as ambiguity-averse, ambiguity-neutral and ambiguity-loving according to the definition of coherent-ambiguity attitude. From this, 43.45% are ambiguity-averse while 22.36% are ambiguity-loving and 18.70% are ambiguity-neural. The 15% (169 subjects) cannot be classified according to the given definition. Specifically, 82 subjects are value-ambiguity averse and choice-ambiguity loving while the remaining 87 are value-ambiguity loving and choice-ambiguity averse. Furthermore, the percentage of classified ambiguity-averse is higher among graduate students than undergraduate ones while ambiguity-neutral prevails in undergraduate students.

Once the aptitude for the ambiguity of individuals was detected, an attempt was made to understand whether this was to some extent related to risk. Parametric and non-parametric tests allow us to observe a significant negative correlation between risk and coherent-ambiguity (p-value ≈ 0). This is coherent with the theory of "uncertainty compensation" [13].

Referring to the pre-established majority thresholds presented in Table 1, we now present the influence ambiguity has on it and if there is a relationship with confidence about how others vote. First of all, we observe that the degree of coherent-ambiguity does not affect the guess about the distribution of voters (p-value = 0.08). In fact, the majority of subjects that exhibit ambiguity aversion (about 74%) have overconfident guesses. Secondly, regardless of the group's size, the majority threshold seems to have a positive (but not so significant) correlation with the degree of ambiguity. This result is confirmed by a Tobit regression too.

4.3 Electoral size and Majority threshold

So far, we have focused on examining the impact of individuals' attitudes towards ambiguity and risk on their choices regarding the majority threshold. However, it is important to consider an additional factor that can influence this relationship: the size of the electoral base. As mentioned earlier, there is a positive relationship between the size of the electorate and the chosen majority threshold. This means that as the number of voters increases, there is a tendency to set a higher majority quorum. We have tried to understand if this happens, according to our model, by considering four groups of different dimensions: a smaller one with lower than 30 participants, two medium-sized ones between 30 and 60 and between

[13] According to this theory people have a certain threshold of tolerance for overall uncertainty, which can be distributed differently between risk and ambiguity. If a person is particularly risk averse, they may prefer situations where the probabilities are clear and definite (known risk) and seek to reduce the ambiguity associated with uncertain outcomes. On the other hand, if a person is more tolerant of ambiguity, they may be willing to deal with situations where the probabilities are uncertain or unknown (high ambiguity) and accept a greater degree of risk.

60 and 100 subjects, and a bigger one with more than 100 voters. A preliminary analysis of the variance (ANOVA) between the chosen quorums and the different dimensions of the electorate gives us a p-value = 0.00978, with a significance level of 0.1%, suggesting the existence of a statistical difference between the groups analyzed. Moreover, a chi-squared test returns a p-value proxy to zero, confirming a significant relationship between the two variables. Figure 1 shows how voters are distributed across the various majority thresholds (from a simple majority to unanimity) as the size of the electoral group increases.

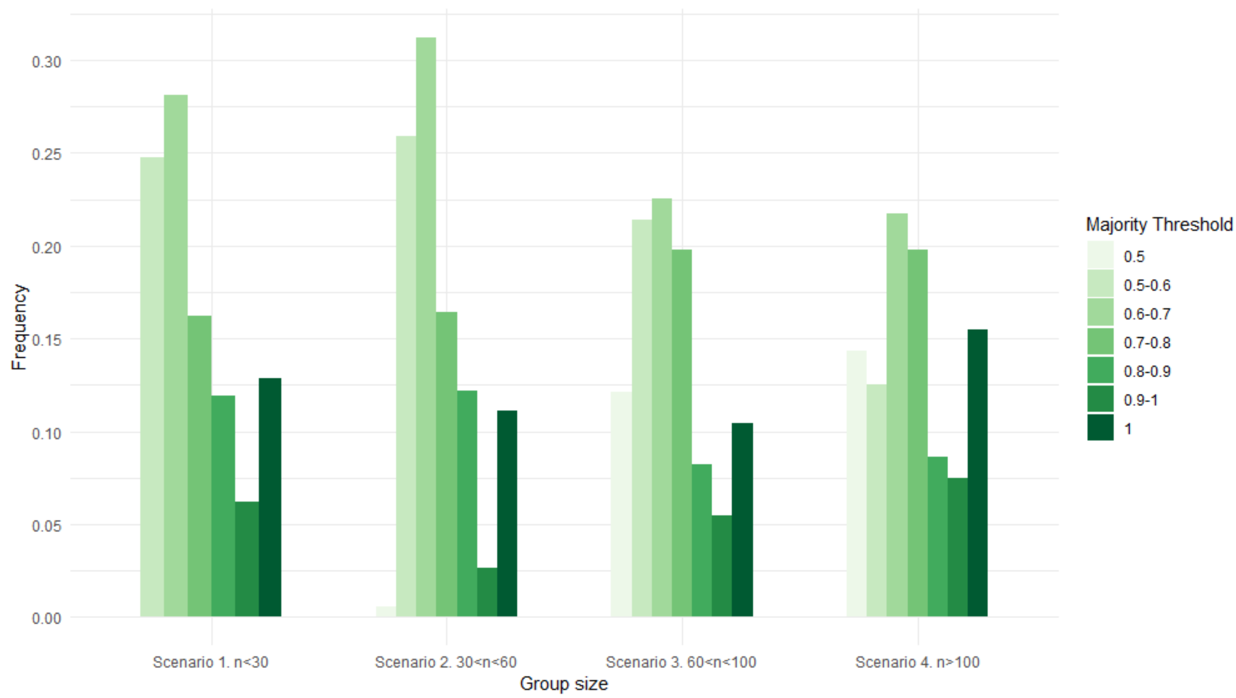

Figure 1: Distribution of majority threshold as electoral size increases

The distribution of votes among the possible majority threshold reflects a desire to ensure fairness, representativeness and mitigate the risks associated with decision-making in a larger and more diverse group. Therefore, the increase in group size influences both risk and ambiguity attitudes, according to the chi-squared test[14]. Figures 2 and 3 report respectively

[14]We have, respectively, for group size - risk a $\chi^2 = 49.56$ and a p-value = 0.0001 and for group size - ambiguity a $\chi^2 = 35.65$ and a p-value = 0.012.

the distribution of the majority threshold for risk and ambiguity across groups.

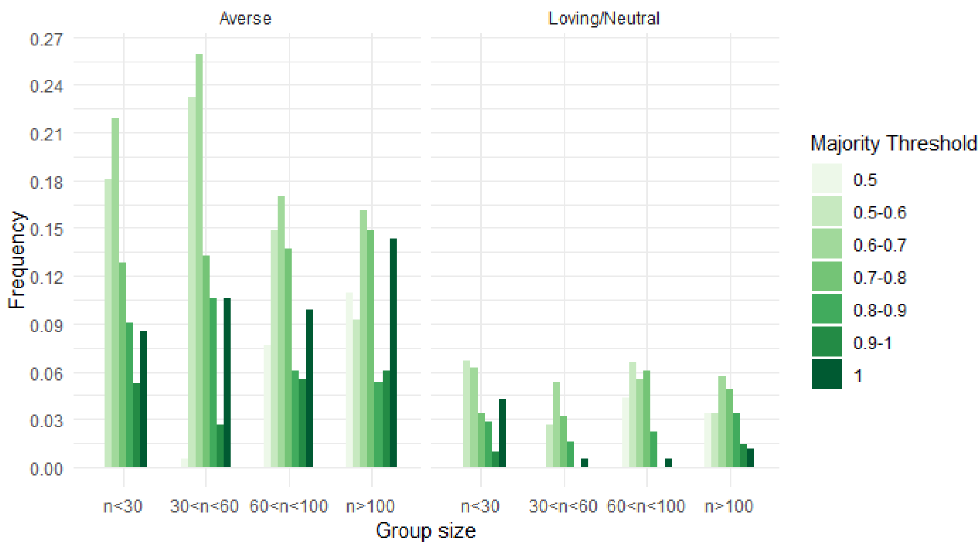

Figure 2: Distribution of majority threshold to electoral size under risk attitude

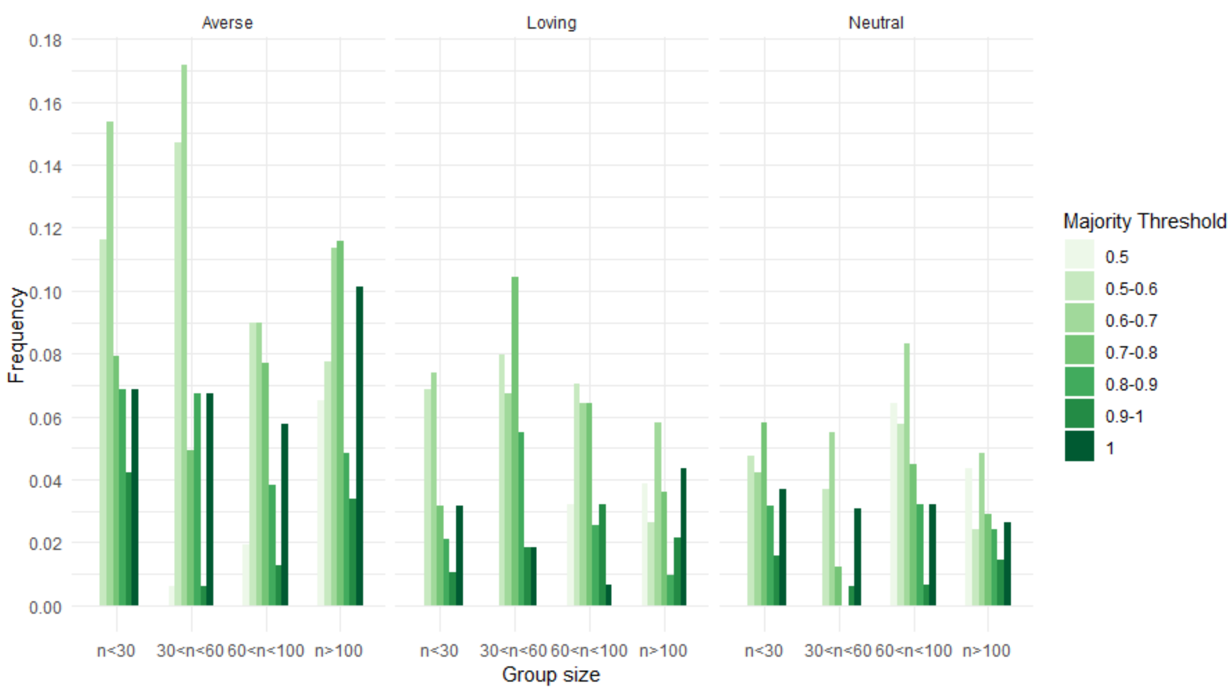

Figure 3: Distribution of majority threshold to electoral size under ambiguity attitude

Observing the behavior of individuals who are averse to ambiguity, we can clearly see that as the size of the group increases, there is a tendency for them to choose a majority threshold that leans towards unanimity more frequently. In fact, there is a significant difference between the increase in ambiguity aversion (decrease of ambiguity loving) and the group size (Kruskal-Wallis, p-value=0.021).

Our findings reveal that as the electorate expands, the impact of ambiguity becomes more pronounced in driving the increase in the sought-after majority threshold. This suggests that the uncertainty and unpredictability associated with ambiguity have a stronger influence on individuals' decisions compared to the traditional risk factor.

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